Table 3 Eigenvalue sensitivities λ_i'

	1st approach	2nd approach	3rd approach
λ_1'	−8.7949159E+3	−8.7949148E+3	−8.828E+3
λ_2', λ_3'	0	0	0
λ_4'	−3.1990803E+3	−3.199075E+6	3.2E+6
λ_5', λ_6'	0	0	0

Table 4 Derivative of the first eigenvector

DOF	1st approach	2nd approach	3rd approach
x_1	0	0	0
y_1	6.7786668E–3	6.7786673E–3	6.78E–3
z_1	0	−2.8604516E–5	−2.5044E–7
ω_{x1}	0	0	0
ω_{y1}	0	5.0429107E–5	−2.6053E–7
ω_{z1}	1.9894528E–2	1.9894527E–2	1.9898E–2
x_2	0	0	0
y_2	3.9641898E–2	3.9641895E–2	3.9648E–2
z_2	0	−8.9063400E–5	−7.7978E–7
ω_{x2}	0	0	0
ω_{y2}	0	6.5473773E–5	5.7324E–7
ω_{z2}	3.9347583E–2	3.9347579E–2	3.9381E–2

Table 5 Effective mass sensitivities

Approach	x	y	z	ω_x	ω_y	ω_z	Mode
1st	0	6.3989737E–1	0	0	0	9.0761532E–1	1st
1st	0	3.6010263E–1	0	0	0	9.2384676E–2	4th
2nd	0	6.3989745E–1	0	0	0	9.0761535E–1	1st
2nd	0	3.6010255E–1	0	0	0	9.2384646E–2	4th
3rd	0	6.4001E–1	0	0	0	9.0777E–1	1st
3rd	0	3.60158E–1	0	0	0	9.2398E–2	4th

amount that eliminates the indeterminacy described in the preceding section. The third approach is the well-known finite difference method, which can be applied by taking a second perturbation (in the example, 10^{-4} kg). Neither the second nor the third approach is exact because the evaluation of the derivative was not performed on the FE model of the given structure but on a perturbed version of it. Furthermore, the third approach has introduced a finite difference approximation. In Tables 3 and 4 the eigenvalue sensitivities and the sensitivity of the first eigenvector, respectively, are reported. Similar results have been obtained for the other eigenvectors. In Table 5 the diagonals of the effective mass matrix sensitivity for the first and fourth modes are reported. All of these results have shown that the second approach had excellent agreement with the exact solution, at least for eigenvalue and effective mass sensitivities, whereas the third one shows acceptable results.

Another possible approach is as follows: 1) determine λ and $\boldsymbol{X}$ from the given structure, 2) determine $\tilde{\boldsymbol{X}}$, $\tilde{\boldsymbol{M}}$, and $\tilde{\boldsymbol{K}}$ from the perturbed one, 3) calculate $\boldsymbol{D}$ from Eq. (12) rewritten at finite difference

$$\boldsymbol{D} = \boldsymbol{X}^T\left(\frac{\tilde{\boldsymbol{K}} - \boldsymbol{K}}{\Delta M_k} - \lambda\frac{\tilde{\boldsymbol{M}} - \boldsymbol{M}}{\Delta M_k}\right)\boldsymbol{X} \tag{14}$$

and 4) find the eigenvalue derivative λ' and $\boldsymbol{\Gamma}$ from Eq. (11). Once $\boldsymbol{\Gamma}$ is determined, one can calculate the eigenvector derivatives with the finite difference approximation

$$\frac{\partial \boldsymbol{X}_{l+i}}{\partial M_k} \approx \frac{\tilde{\boldsymbol{X}}_{l+i} - \boldsymbol{X}_{l+i}\boldsymbol{\Gamma}}{\Delta M_k} = \frac{\tilde{\boldsymbol{X}}_{l+i} - \boldsymbol{Z}_i}{\Delta M_k} \qquad i = 1, \ldots, m \tag{15}$$

The results obtained were very close to the results of the third approach.

Conclusions

A generalization for the calculation of effective mass sensitivities to the case of coincident eigenvalues has been proposed. The results obtained with the exact approach and an approach based on a suitable perturbation, introduced to eliminate the eigenvalue multiplicity, are in excellent agreement. The two approaches based on the finite difference approximation are still in good agreement, but as expected, the elimination of significant digits does affect the precision.

References

[1]Wada, B. K., Bamford, R., and Garba, J. A., "Equivalent Spring-Mass System: a Physical Interpretation," *Shock and Vibration Bulletin*, Vol. 4, Jan. 1972, pp. 215–225.

[2]Sambasiva Rao, M., Nair, P. S., and Durvasula, S., "Equivalent Dynamic Models for a Spacecraft and its Subsystems," *ESA Journal*, Vol. 10, No. 1, 1986, pp. 93–111.

[3]Chung, Y. T., and Sernaker, M. L., "Assessment of Target Mode Selection Criteria for Payload Modal Survey," *Proceedings of the Twelfth International Modal Analysis Conference* (Honolulu, HI), Society for Experimental Mechanics, Bethel, CT, 1994, pp. 272–279.

[4]Paolozzi, A., Peroni, I., and Ciccolini, P., "Effective Mass Sensitivities in Structural Dynamics Modification," *Proceedings of the Thirteenth International Modal Analysis Conference* (Nashville, TN), Society for Experimental Mechanics, Bethel, CT, 1995, pp. 1490–1498.

[5]Paolozzi, A., and Peroni, I., "A Procedure for the Determination of Effective Mass Sensitivity in a General Tridimensional Structure," *Computers and Structures*, Vol. 62, No. 6, 1997, pp. 1013–1024.

[6]Imbert, J. F., *Analyse des Structures par Elements Finis*, Cepadues Edition, Toulouse, France, 1984, pp. 414–438.

[7]Ojalvo, I. U., "Efficient Computation of Modal Sensitivities for Systems with Repeated Frequencies," *AIAA Journal*, Vol. 26, No. 3, 1988, pp. 361–366.

[8]Dailey, R. L., "Eigenvector Derivatives with Repeated Eigenvalues," *AIAA Journal*, Vol. 27, No. 4, 1989, pp. 486–491.

[9]Fox, R. L., and Kapoor, M. P., "Rates of Change of Eigenvalues and Eigenvectors," *AIAA Journal*, Vol. 6, No. 1, 1968, pp. 2426–2429.

A. Berman
Associate Editor

Modal Data Are Insufficient for Identification of Both Mass and Stiffness Matrices

Menahem Baruch*
Technion—Israel Institute of Technology, Haifa 32000, Israel

Introduction

IN the technical literature, there have appeared, and continue to appear, many papers in which the researchers propose to identify simultaneously both the mass and stiffness matrices of a dynamic structure by applying only modal measured data. However, it can be shown that even full modal data are insufficient for the identification of both the mass and the stiffness matrices.[1,2] In Refs. 3 and 4, it was proposed to use the measured mode shapes as a reference basis in the process of correction of the mass and the stiffness matrices of a structure. The problem is that the mode shapes are not uniquely defined. Any mode shape can be multiplied by a nonzero constant without changing its physical meaning. In Ref. 5, Huang and Craig, who dealt with a six-degree-of-freedomstructure, wrote, "It should be noted that even when all six modes are used, the correct values of the mass and stiffness matrices cannot be obtained." The reason for this is that, for given mode shapes and natural frequencies, the mass and stiffness matrices are not uniquely defined. We will now show that the same mode shapes and natural frequencies can be obtained for an infinite number of different pairs of stiffness and mass matrices.

Received July 23, 1997; accepted for publication Aug. 18, 1997.

*Professor Emeritus, Faculty of Aerospace Engineering, Technion City.

Infinite Number of Solutions

Let $X(n \times n)$ and $\Omega^2(n \times n)$ be the modal and frequency matrices, respectively, that satisfy the frequency equation of a structure with n degrees of freedom:

$$KX = MX\Omega^2 \tag{1}$$

where $M(n \times n)$ and $K(n \times n)$ are the mass and the stiffness matrices of a given dynamic structure, respectively.

The modal matrix can be normalized to obtain

$$X^t MX = I \qquad X^t KX = \Omega^2 \tag{2}$$

where $(\cdot)^t$ represents the transpose of a matrix and $I(n \times n)$ represents the unit matrix. Any one of the mode shapes can be multiplied by an arbitrary constant different from zero, and it will still represent the same mode shape. Hence,

$$\phi = Xd \tag{3}$$

where $d(n \times n)$ is an arbitrary nonsingular diagonal matrix and $\phi(n \times n)$ also represents the modal matrix of the structure. Clearly, ϕ satisfies the frequency equation, Eq. (1). We will assume that there exists another pair of mass and stiffness matrices that yield the same modal and frequency matrices,

$$\phi^t \bar{M}\phi = I \qquad \phi^t \bar{K}\phi = \Omega^2 \tag{4}$$

from which one obtains

$$\begin{aligned} \bar{M} &= (\phi\phi^t)^{-1} = (Xd^2X^t)^{-1} \\ \bar{K} &= \phi^{-t}\Omega^2\phi^{-1} = (dX^t)^{-1}\Omega^2(Xd)^{-1} \end{aligned} \tag{5}$$

where $(\cdot)^{-1}$ represents the inverse of a matrix. Because the matrix d is arbitrary, it is clear that there exists an infinite number of pairs of mass and stiffness matrices that have the same modal and frequency matrices. Hence, the mode shapes cannot be a reference basis for the identification of a structure.

In other words, by measuring the natural frequencies only, or even measuring the natural frequencies and the mode shapes, one cannot identify in a unique way both the stiffness and the mass matrices. As said before, the authors of many papers have considered the mode shapes as a reference basis for the simultaneous identification of the stiffness and mass matrices. As proved in Ref. 1 and as shown, for example, in Ref. 5, the matrices identified in this way may be quite different from the actual stiffness and mass matrices.

Conclusion

Simultaneous changes in the mass and stiffness matrices cannot be identified by using modal data only. The mode shapes cannot provide a reference basis.

References

[1] Baruch, M., "Mass Change Detection Based on Reduced Measurements," Faculty of Aerospace Engineering, Technion—Israel Inst. of Technology, TAE Rept. 761, Haifa, Israel, Nov. 1995.

[2] Baruch, M., "Simultaneous Stiffness and Mass Change Detection Based on Reduced Measurements," Faculty of Aerospace Engineering, Technion—Israel Inst. of Technology, TAE Rept. 780, Haifa, Israel, June 1996; also American Society of Mechanical Engineers, Symposium on Model Updating, DETC97/VIB-4143, Sacramento, CA, Sept. 1997.

[3] Berman, A., "Mass Matrix Correction Using an Incomplete Set of Measured Modes," *AIAA Journal*, Vol. 17, No. 10, 1979, pp. 1147, 1148.

[4] Baruch, M., "Optimal Correction of Mass and Stiffness Matrices Using Measured Modes," *AIAA Journal*, Vol. 20, No. 11, 1982, pp. 1623–1626.

[5] Huang, H. H., and Craig, R. R., Jr., "Model Updating Based on Sensitivities of System Matrices," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 34th Structures, Structural Dynamics, and Materials Conference* (La Jolla, CA), AIAA, Washington, DC, 1993, pp. 3041–3049.

A. Berman
Associate Editor